

# Why do all these primitives defined over rings/prime fields get broken?

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SPRING/EC'26/CAHF



## Full Round Attacks

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What happened? **Should we PANIC?!??**

## Outline

- 1 Full round attacks
- 2 These primitives are harder to design
- 3 Causes of death
- 4 Conclusion

## Plan of this Section

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  - On designing a symmetric primitive
    - New use cases: the need for arithmetizations
    - STAP Examples
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## A Question

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Security · Efficiency

## Unstable Definitions

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- What are the operations that we **can** use?
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How to get the best security for a given price?

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(We'll get back to that later)

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## Securing Computation

More and more protocols intend to secure **computations**.

**FHE** Fully Homomorphic Encryption

**MPC** Multi Party Computations

**ZK-\*** Zero Knowledge- [ proof, argument... ]

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One Approach to Rule Them All (?): **arithmetization**

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Verifying if  $y = c(ax + b)^{10} + x$  in R1CS

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2  $t_1 = t_0 + b$

3  $t_2 = t_1 \times t_1$

4  $t_3 = t_2 \times t_2$

5  $t_4 = t_3 \times t_3$

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- How to turn a computation into an arithmetic circuit depends on the operations allowed
- Its cost is also arithmetization-dependent—**though low degree is usually welcome!**

## A not basic at all example of arithmetization

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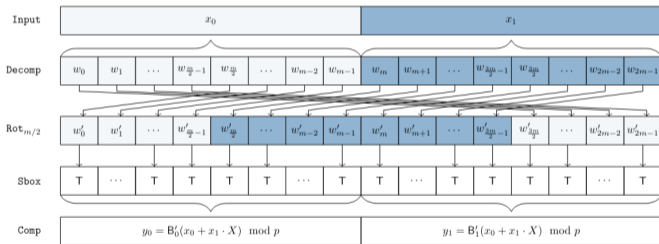


Figure 3: The Bar layer  $B' : \mathbb{F}_{p^n} \rightarrow \mathbb{F}_{p^n}$  for  $n = 2$  in detail, including the decomposition, the rotation, the S-box, and the composition.

source: *Skyscraper: Fast Hashing on Big Primes*,  
<https://eprint.iacr.org/2025/058.pdf>

## Symmetric Techniques for Advanced Protocols

MPC

FHE

ZK

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Masking

BGV

R1CS

MPC-in-the-head  
(signatures...)

FV

AIR  
...

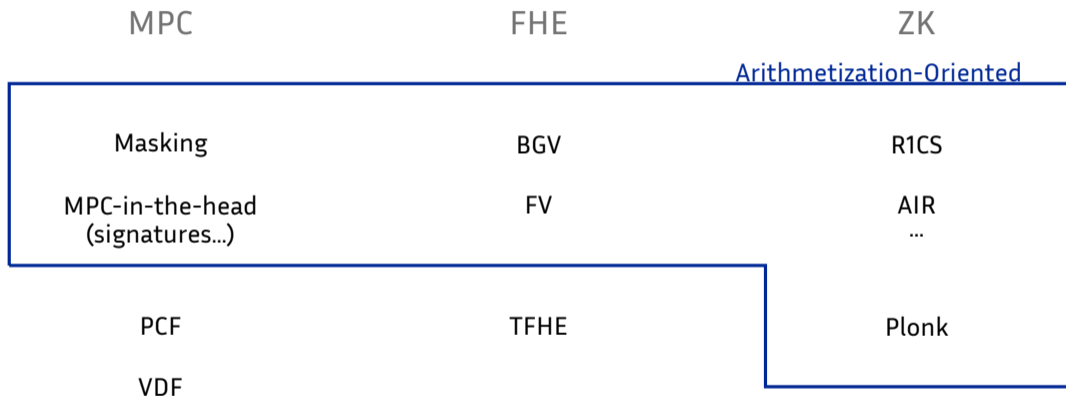
PCF

TFHE

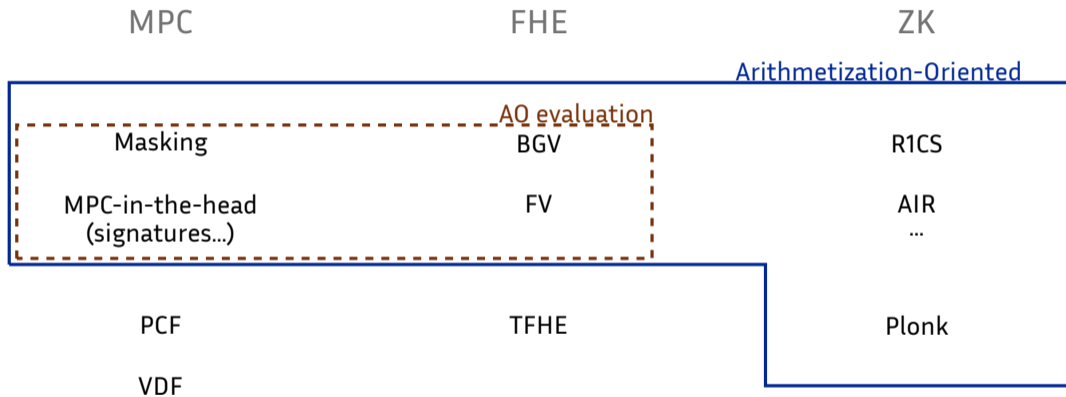
Plonk

VDF

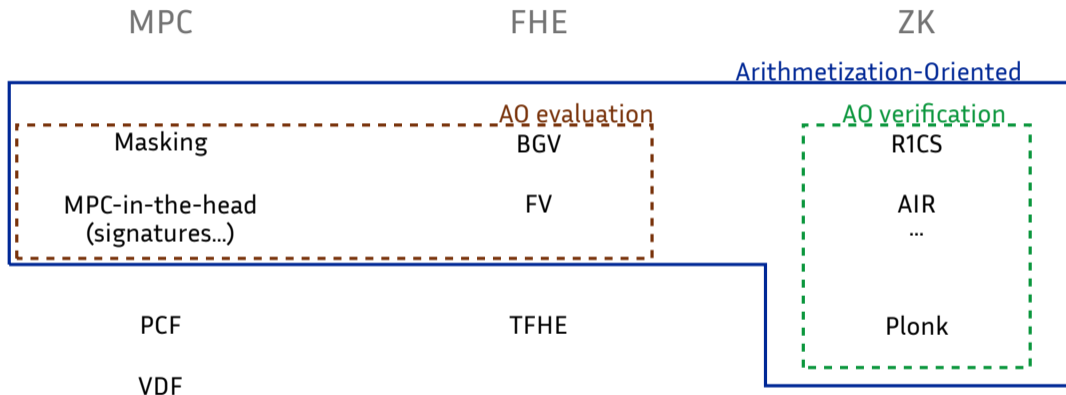
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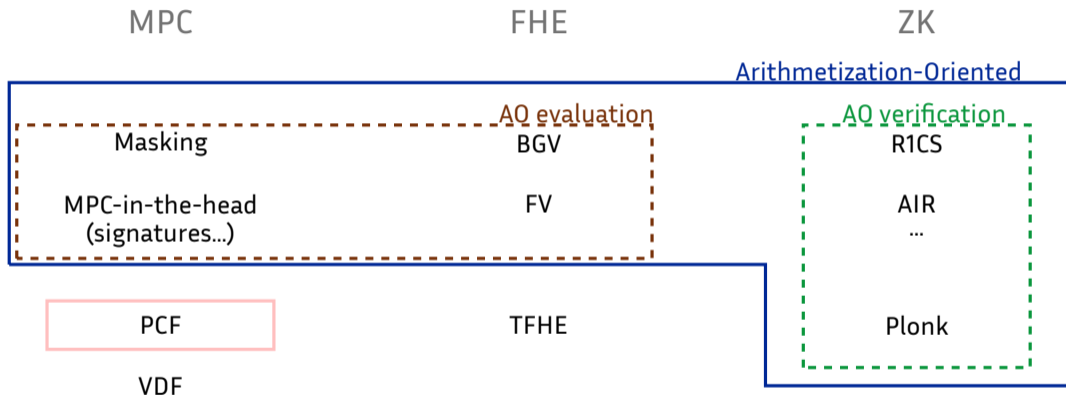
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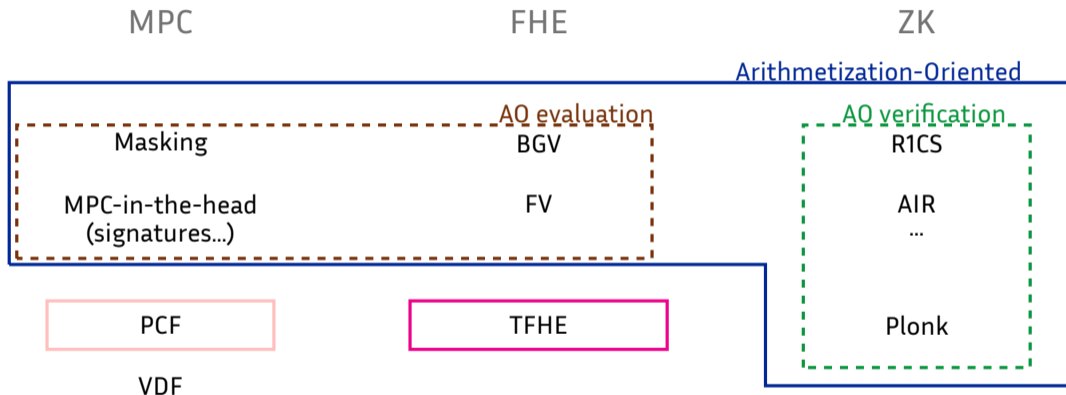
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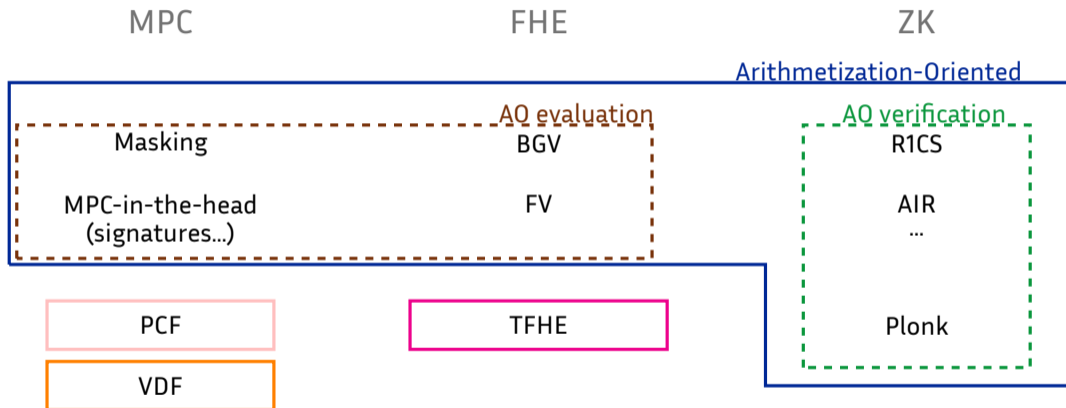
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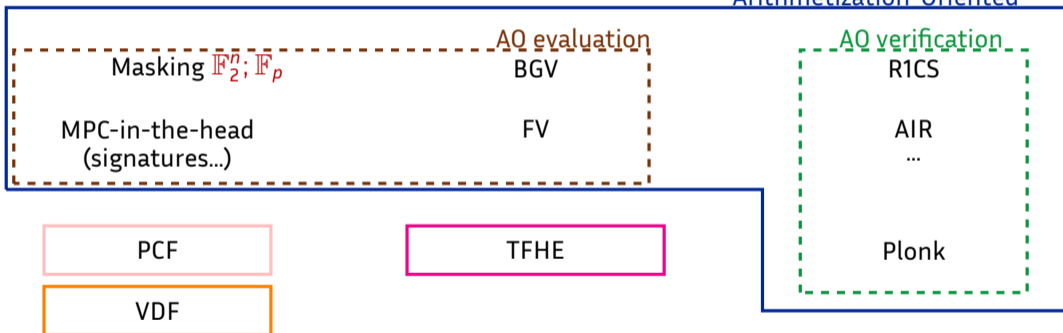
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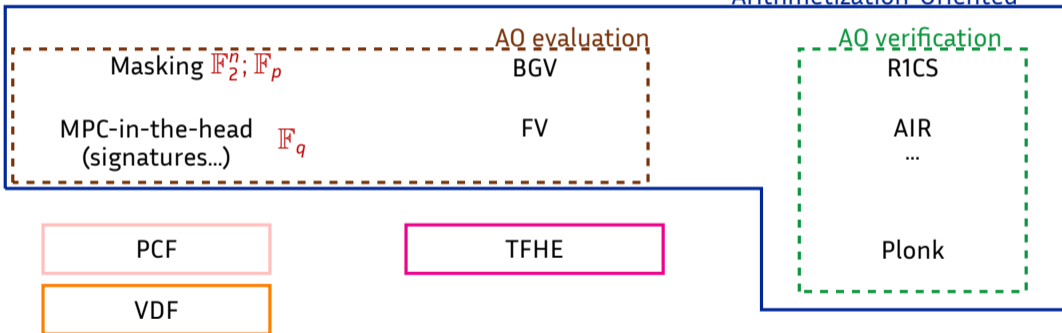
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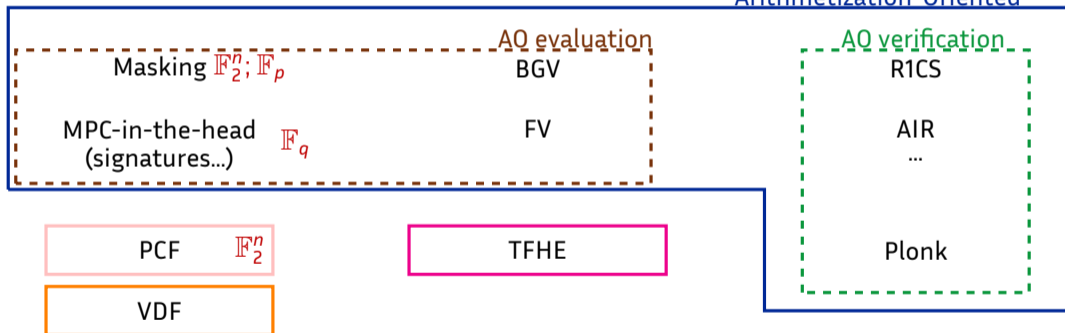
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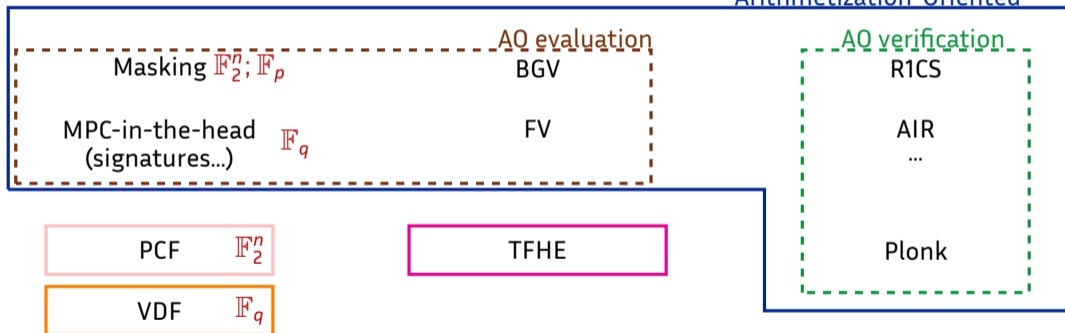
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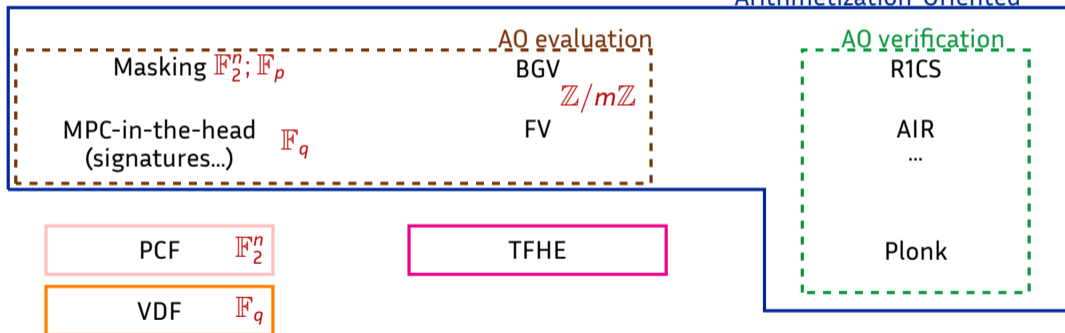
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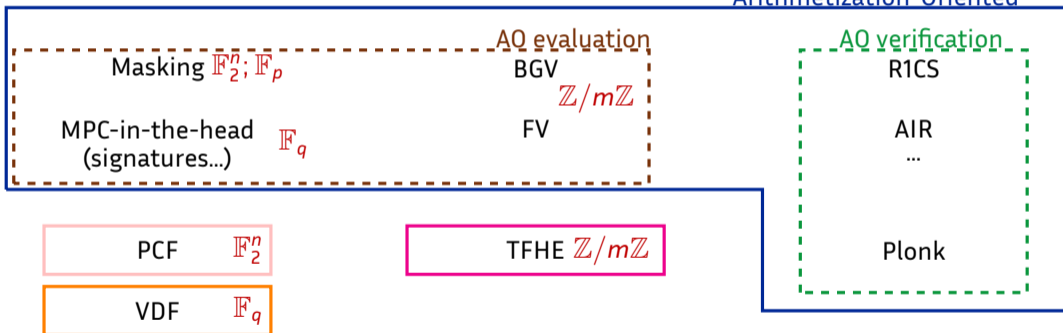
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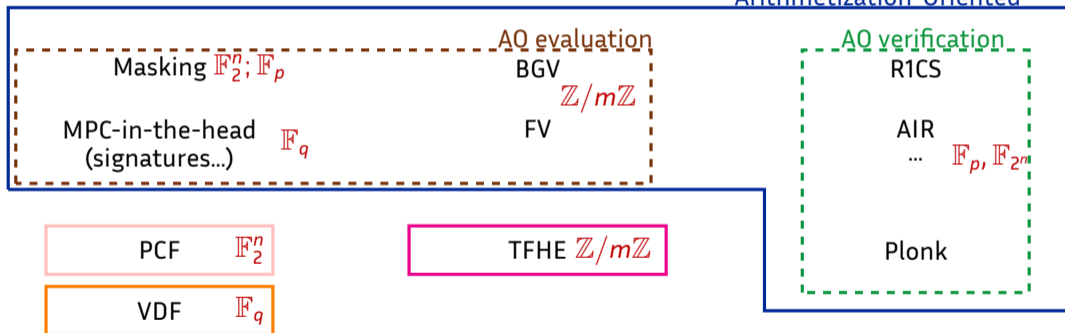
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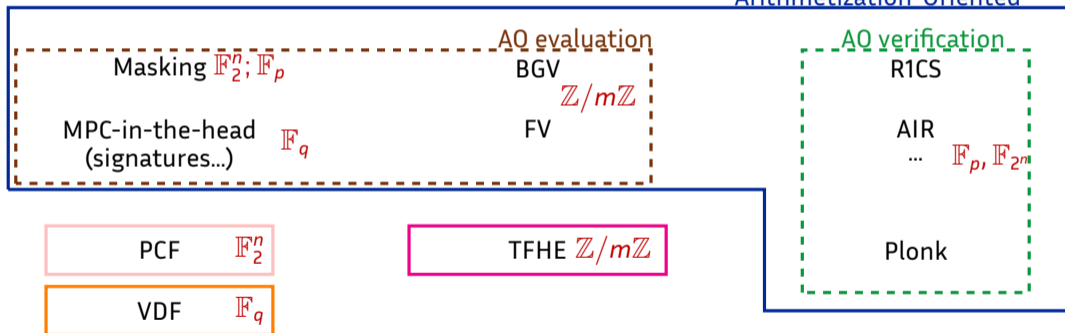
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It's kind of a mess!

## Du passé, faisons table rase (?)

---

**Old World**

**New World**

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Design strategy	Prevent diff./integral attacks → wide trail strategy	???

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Elisabeth-4  $q = 2^4$

Uses a constant key register on which index-dependent non-linear functions are applied.

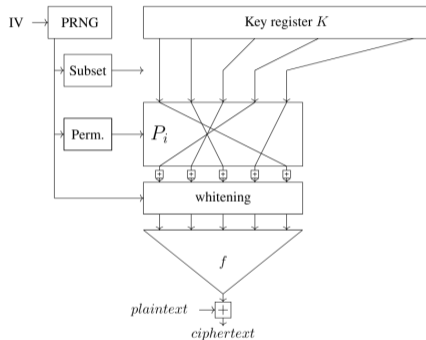


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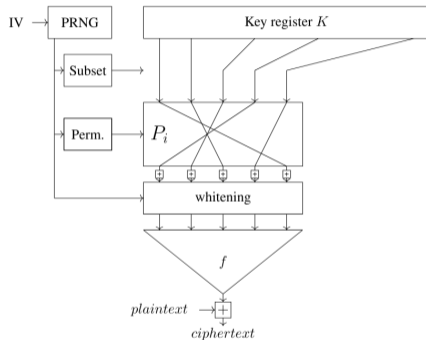


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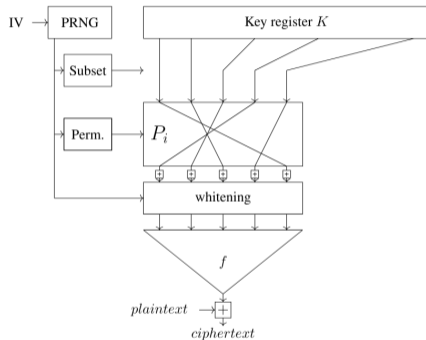


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**Transistor**  $q = 2^4 + 1$   
SNOW-like round structure

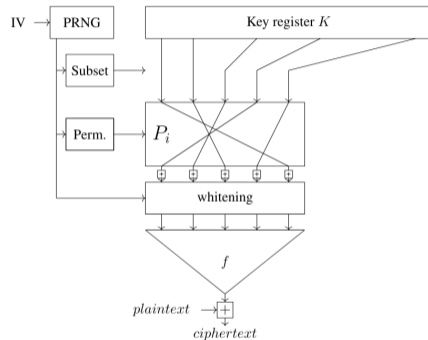


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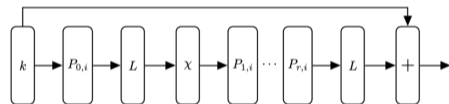


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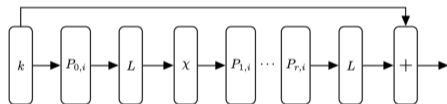


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- HERA  $q$  large prime  
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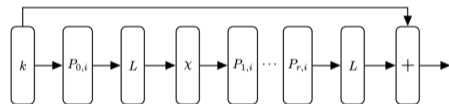


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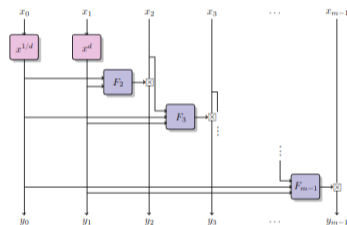
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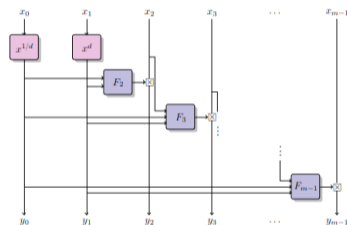
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- Anemoi**  $q = 2^n$  or large prime  
Uses the “Flystel”, a high degree S-box  
*CCZ-equivalent* to a function of low degree.



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  - A graveyard?
  - PoSSo-Based Cryptanalysis
  - The even-less-simple case
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## “Broken” Primitives

Use	Name	Security (intended)	Best Attack	
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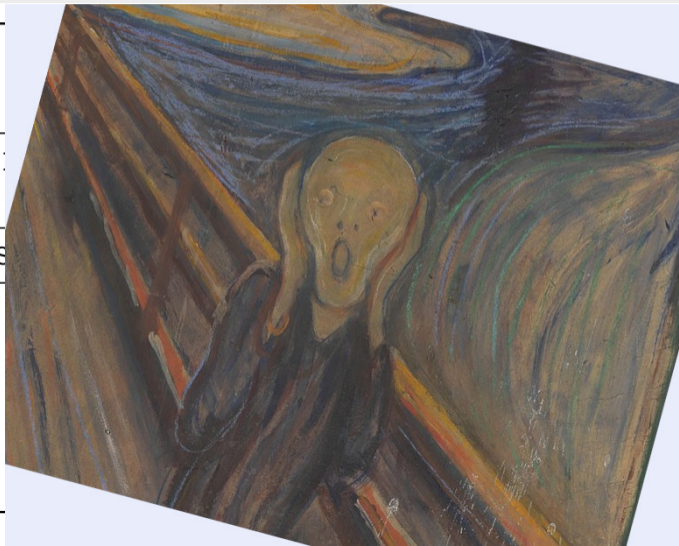
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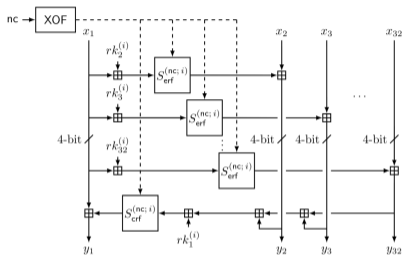
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ZK	Jarvis	128	83	ad hoc PoSSo
	Anemoui	128	80	<b>Resultants</b>
	Arion	128	90	
	Griffin	128	55	
	Rescue	512	475	

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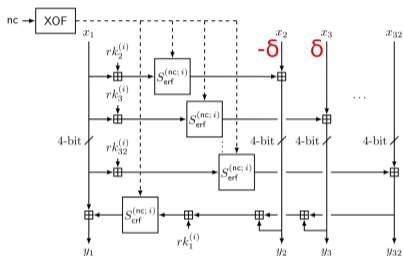


## An appetizer: The case of FRAST



**Figure 2:** The  $i$ -th round function of FRAST.  $r_{k_1}^{(i)}, \dots, r_{k_{32}}^{(i)}$  are the  $i$ -th round keys.

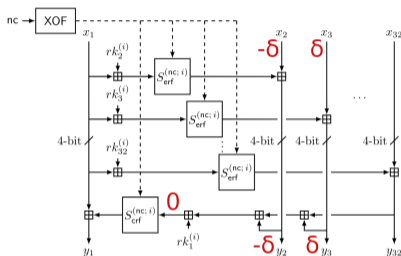
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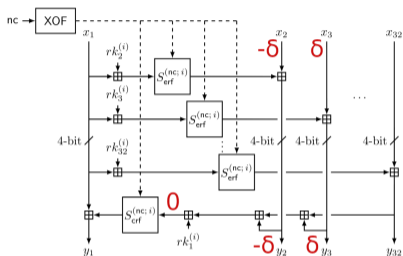


Figure 2: The  $i$ -th round function of FRAST.  $rk_1^{(i)}, \dots, rk_{32}^{(i)}$  are the  $i$ -th round keys.

- 1 Proba 1 iterated differential:  $(0, -\delta, \delta, 0, \dots, 0)$
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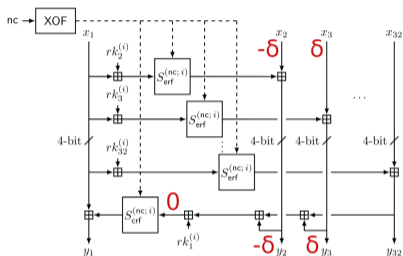


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### Almost Affinity

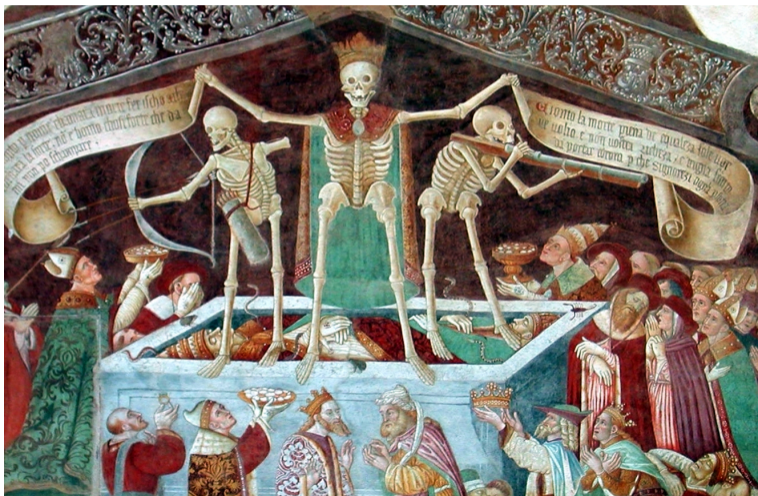
Let  $E_K$  be the internal block cipher of FRAST

$$\Pr_{\Delta} [\forall x, E_K(x + \Delta) = E_K(x) + \Delta] = 2^{-8}$$

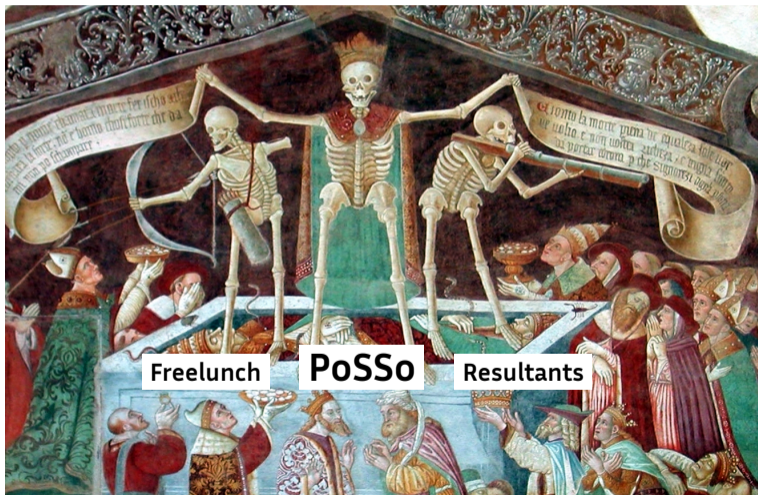
## Plan of this Section

- 1 Full round attacks
- 2 These primitives are harder to design
- 3 Causes of death**
  - A graveyard?
  - PoSSo-Based Cryptanalysis**
  - The even-less-simple case
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## Allegory



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## Definition of PoSSo

### Polynomial System Solving

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Here, we suppose that there is a finite, small number of solutions (e.g. not a full vector space).

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## Steps of a PoSSo-Based Attack

**Initial Cryptanalysis.** Deduce a system of equations **as simple as possible** from the intended attack.

**Write a system of equations.**

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## The particular case of Elisabeth (1/2)

### Elisabeth

Implements a “filter permutator”

- Key register is never modified
- Optimized for TFHE
- Low multiplicative depth:  $F$  is of low degree

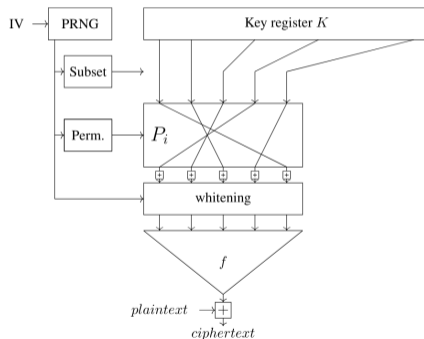


Fig. 1: The group filter permutator design

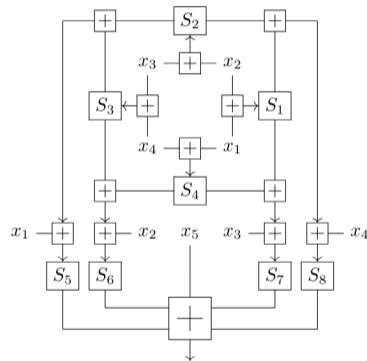


Fig. 2: Elisabeth-4's 5 to 1 inner function.

## The particular case of Elisabeth (2/2)

### Basic Linearization

$$\begin{aligned} s_i &= F(x_0, \dots, x_n) \\ &= \sum_{u \in \mathbb{F}_2^n} \alpha_u \prod_j x_j^{u_j} = \sum_{u \in \mathbb{F}_2^n} \alpha_u P_u \end{aligned}$$

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Model	Time (operations)	Data (nibbles)	Memory (bits)
Known IV	$2^{124}$	$2^{43}$	$2^{87}$
Known IV	$2^{116}$	$2^{41}$	$2^{81}$
Known IV	$2^{94}$	$2^{41}$	$2^{57}$
Known IV	$2^{88}$	$2^{87}$	$2^{54}$
Chosen IV	$2^{88}$	$2^{37}$	$2^{54}$

## The simplest case: linearization

would be more costly in this context. On a filtered LFSR the security estimation is  $\mathcal{O}(D \log^2(D) + ED \log(D) + E^\omega)$  where  $D = \sum_{i=1}^{\deg(h)} \binom{N}{i}$  and  $E = \sum_{i=1}^{\deg(g)} \binom{N}{i}$ . This estimation will be used as an indicator rather than a sharp limit, considering that the complexity of the best attack of the algebraic kind would lie between this (too low) bound and the (too high) one given by the algebraic attack of Courtois-Meier.

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The best algorithm was **not the one considered**, and its complexity **scaled differently**.

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- 1 Free Gröbner basis: the **FreeLunch**
- 2 Compute  $M_{x_0}$ : regular Gröbner basis arithmetic
- 3 Solve  $\det(x_0 I - M_{x_0}) = 0$ : **dedicated algorithm**

## The Resultant-based Approach

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$\underbrace{\hspace{10em}}_{\gamma+\delta}$

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## Towards better security arguments

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**All is not lost:** univariate root finding is always there, so

*$D_I$  is<sup>a</sup> a good metric!*

---

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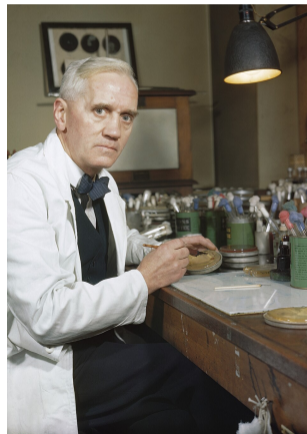
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**All is not lost:** univariate root finding is always there, so

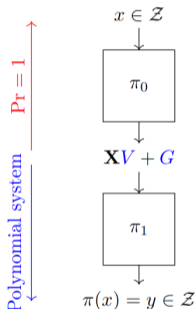
*$D_1$  is<sup>a</sup> a good metric!*

---

<sup>a</sup>Hopefully...



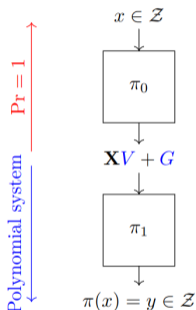
## Round Skips: a limit of $D_I$ ...



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Use the degrees of freedom to add equations that decrease the ideal degree.

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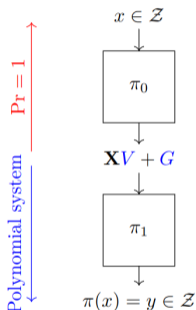


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Bigger blocks  $\rightarrow$  higher  $D_I$ , **but also** more degrees freedom!

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- 1 Full round attacks
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$\alpha$	3	5	7	11
$\ell = 1$	21	21	20	19
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$\ell = 3$	12	12	12	11
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$\ell = 6$	10	10	10	10
$\ell = 8$	10	10	9	9

(a) When  $s = 128$ .

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$\ell = 1$	37	37	36	35
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$\ell = 3$	17	17	17	17
$\ell = 4$	16	16	15	15
$\ell = 6$	13	13	13	13
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**Comparison  $\neq$  Security evaluation** Cryptanalysts focus on **comparing instances**

Is there a set of parameters for which my attack works?

$\neq$

What is the security of the variant I care about?

## Looking at Rescue

This slide and the following ones are heavily based on

**Improved Resultant Attack against Arithmetization-Oriented Primitives**, Bariant et al.

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$r$	iteRes	uniSol	[26]	Designers' estimation [23]
6	49	40	60	-
7	57	45	69	-
8	64	50	81	-
12	93	70	-	80
18	137	99	-	128
35	257	181	-	256
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**Table 3.** Theoretical complexities (in  $\log_2 \mathbb{F}_q$ -multiplications) for attacking variants of Rescue with  $t = \alpha = 3$  and  $\log_2(q) \approx 512$ . Our analysis show that the complexity of `sysGen` is negligible compared to the rest of the attack.

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All instance are fine except  
 for **a single one** (the 512-bit  
 one)

## Looking at Anemoi

$\alpha$	$\ell$	$r$	sysGen	iteRes	[3, polyDet]	[26]	$\alpha$	$\ell$	$r$	sysGen	iteRes	[3, polyDet]	[26]
3	1	21	78	80	118	110	3	1	37	129	130	203	191
	2	14	89	89	-	-		2	22	133	134	-	-
	3	12	105	104	-	-		3	17	144	144	-	-
	4	12	131	131	-	-		4	16	172	172	-	-
	6	10	154	153	-	-		6	13	199	198	-	-
5	1	21	91	96	156	133	5	1	37	151	157	270	231
	2	14	108	113	-	-		2	22	164	169	-	-
	3	12	131	135	-	-		3	17	182	186	-	-
	4	12	167	171	-	-		4	16	221	224	-	-
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	6	10	264	270	-	-		6	13	345	351	-	-

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- Bigger  $\alpha$   $\rightarrow$  more secure

When these parameters increase, other attacks take over (in the initial security analysis)

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## Potential directions for hardening

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**Thank You!**

## Looking at Arion

<i>e</i>	<i>t</i>	<i>r</i>	sysGen	iteRes	uniSol	[3, polyDet]	<i>e</i>	<i>t</i>	<i>r</i>	sysGen	iteRes	uniSol	[3, polyDet]
3	3	6	75	90	73	128	3	3	5	63	78	62	104
4	6	82	96	79	134		4	4	55	69	54	84	
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6	5	78	91	75	119		6	4	62	76	61	92	
8	4	69	82	67	98		8	4	69	82	67	98	
5	3	6	79	94	76	132	5	3	4	53	68	53	83
4	5	72	86	70	113		4	4	57	72	57	87	
5	5	76	90	74	118		5	4	61	75	60	91	
6	5	81	95	78	122		6	4	65	79	64	94	
8	4	71	85	70	101		8	4	71	85	70	101	

**Table 2.** Theoretical complexities (in  $\log_2 \mathbb{F}_q$ -multiplications) of attacks against full-round variants of Arion with 128-bit security using  $\alpha = 121$ . Normal parameters are in the left table, and the more aggressive  $\alpha$ -Arion parameters are in the right table.

## Looking at Griffin

$\alpha$	$t$	$r$	sysGen	iteRes	uniSol	[3, polyDet]
3	3	16	92	92	83	120
	4	15	87	87	78	112
	8	11	63	61	57	76
	$\geq 12$	10	55	53	51	64
5	3	14	102	106	92	141
	4	11	81	85	75	110
	8	9	63	66	58	81
	$\geq 12$	9	60	62	56	74

**Table 1.** Complexity (in  $\log_2 \mathbb{F}_q$ -multiplications) of the steps of our attack on full-round variants of Griffin with 128-bit security claim.